

Modeling Transportation Network Flows as a Simultaneous Function of Travel Demand, Earthquake Damage, and Network Level Service.

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Abstract

This paper summarizes the results of a transportation analysis incorporating a novel approach to modeling variable transportation demand. This approach links the demand for destinations to the level of service available on the transportation network. The paper includes applications to a set of predicted, post-earthquake transportation network configurations for the city of Memphis, Tennessee, and to network damage resulting from a large scenario earthquake in the San Francisco Bay Area.

Introduction

In addition to replacement and repair cost of damage to transportation structures, large earthquakes produce an increase in time delay resulting from these network components' loss of function. Earthquake losses due to travel time increases may be evaluated by examining the difference between baseline conditions and network performance following a scenario earthquake.

Theoretical Background of Variable Demand Model.

Competitive Flows in Transportation Networks. The user equilibrium network flow model is one of the most useful transportation analysis models. In a standard, fixed demand model, total demand for travel between an origin and destination (OD) pair does not vary with travel cost between the origin and destination. Route selection is a function of route delay, but the propensity to travel is not.

More realistically, trip rates are influenced by the level of service available on the transportation network. Following a large earthquake, the congestion level would increase because of reductions in network capacity. The increased congestion level would then induce a reduction in travel demand. Figure 1 shows this relationship. Before the earthquake, the transportation system supplies service according to the function S_1 , and flows d_1 use the system at cost of p_1 . After the earthquake damages the network, the level of service drops to S_2 , and demand for travel decreases in

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response to this drop in level of service. Flows d_2 , occur at (delay) cost of p_2' . This effect can be accounted by an appropriate function relating trip rates to travel times.

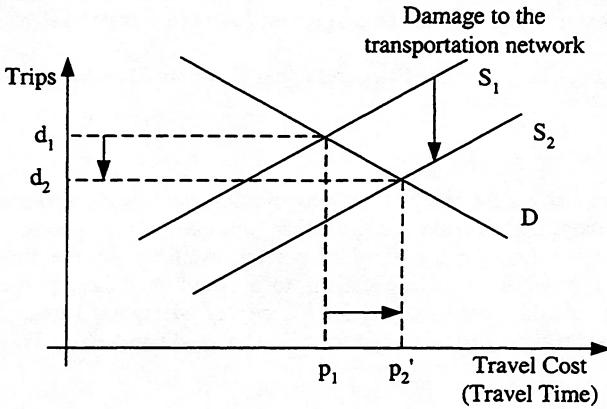


Figure 1. Transportation demand, supply, and user equilibrium delay costs in a network with reduced capacity and variable demand for travel: Movement along the travel demand curve.

User equilibrium flows subject to variable demand for travel occur when the travel times on all used paths between any pair of origin-destination zones are equal, and are also equal to or less than the travel times on any unused paths connecting this pair. In addition, the trip requirements for all of the OD pairs in the network should conform to the travel demand function. The problem addressed in this paper is that of finding a set of link volumes, link travel times, and OD requirements that simultaneously satisfy the conditions for user equilibrium on the transportation network. The mathematical form of the static, path flow version of this problem is (Beckmann, McGuire, and Winston, 1956),

$$\max z(\mathbf{x}, \mathbf{q}) = \sum_a \int_0^{x_a} t_a(w) dw - \sum_r \int_0^{q_r} D_r^{-1}(w) dw \tag{1}$$

subject to

$$\sum_k f_k^{rs} = q_{rs} \quad \forall r, s \tag{2}$$

$$f_k^{rs} \geq 0 \quad \forall k, r, s \tag{3}$$

$$q_{rs} \geq 0 \quad \forall r, s \tag{4}$$

$$q_{rs} = D_{rs}(u_{rs}) \quad \forall r, s \quad (5)$$

$$x_a = \sum_{rs} \sum_k f_k^{rs} \cdot \delta_{a,k}^{rs} \quad \forall a \quad (6)$$

where

- t_a = link performance function of link a ,
- D = travel demand function,
- D^{-1} = inverse of the travel demand function,
- f_k^{rs} = flow on path k connecting OD pair $r-s$,
- q_{rs} = trip rate between OD pair $r-s$,
- u_{rs} = travel time between OD pair $r-s$,
- x_a = flow on link a , and
- $\delta_{a,k}^{rs}$ = 1 if link a is on path k between OD pair $r-s$,
0 otherwise.

The first term on right-hand side of Equation (1) ensures that link volumes and travel times meet user equilibrium conditions. The second term adjusts trip rates between OD pairs so that the travel demand loaded on to network is an appropriate function of travel time.

Solution Procedure. We developed a numerical algorithm based on the secant method to solve this highly endogenous nonlinear programming problem. The algorithm is summarized as follows.

Step 0: Initialization.

Find an initial feasible flow pattern $\{x_a^n\}, \{q_{rs}^n\}$. Set iteration counter $n:=1$.

Step 1: Update link travel times as a function of the feasible set of link flows update and the equilibrium travel times as a function of trip making requirements.

$$\text{Set } t_a^n = t_a(x_a^n) \forall a; \text{ compute } D_{rs}^{-1}(q_{rs}^n) \forall r, s. \quad (7)$$

Step 2: Find auxiliary link volumes and trip rates.

Compute the shortest path, m , between each O-D pair $r-s$ based on link travel time $\{t_a^n\}$, $c_m^{rs^n} = \min_{\forall k} \{c_k^{rs^n}(t_a^n)\}$. (8)

Find auxiliary trip rates.

If $c_m^{rs^n} < D_{rs}^{-1}(q_{rs}^n)$, set $g_m^{rs^n} = \overline{q_{rs}^n}$ where m is shortest path, and $\overline{q_{rs}^n}$ is the upper bound on the trip rate.

$$\text{If } c_m^{rs^n} > D_{rs}^{-1}(q_{rs}^n), \text{ set } g_k^{rs^n} = 0 \forall k. \quad (9)$$

$$\text{If } \left| c_m^{rs} - D_{rs}^{-1}(q_{rs}^n) \right| < \varepsilon, \text{ set } g_m^{rs} = g_m^{rs^{n-1}}. \quad (10)$$

$$\text{Compute auxiliary link volumes } y_a^n = \sum_{rs} \sum_k g_k^{rs} \cdot \delta_{a,k}^{rs} \quad \forall a. \quad (11)$$

$$\text{Compute auxiliary trip rates } v_{rs}^n = \sum_k g_k^{rs} \quad \forall r, s. \quad (12)$$

Step 3: Find the best direction of search.

Solve following system for α .

$$\min z(\alpha) \sum_a \int_0^{x_a^n + \alpha(y_a^n - x_a^n)} t_a(w) dw - \sum_{rs} \int_0^{q_{rs}^n + \alpha(v_{rs}^n - q_{rs}^n)} D_{rs}^{-1}(w) dw \quad (13)$$

$$\text{subject to } \quad 0 \leq \alpha \leq 1$$

Step 4: Update the feasible set of link flows.

$$x_a^{n+1} = x_a^n + \alpha_n (y_a^n - x_a^n) \quad (14)$$

$$q_{rs}^{n+1} = q_{rs}^n + \alpha_n (v_{rs}^n - q_{rs}^n) \quad (15)$$

Step 5: Test for convergence.

If the following inequality holds for sufficiently small κ , stop. Otherwise, set iteration counter $n := n+1$ and go to step 1.

$$\sum_{rs} \frac{|D_{rs}^{-1}(q_{rs}^n) - u_{rs}^n|}{u_{rs}^n} + \sum_{rs} \frac{|u_{rs}^n - u_{rs}^{n-1}|}{u_{rs}^n} \leq \kappa \quad (16)$$

This algorithm is an extension of the standard Frank-Wolfe algorithm used to solve the fixed-demand version of the user equilibrium problem. The key extension is the auxiliary trip rates found in Step 2 (Sheffi 1985).

Calibrating the Travel Demand Function. The travel demand function might be expected to be strictly decreasing with respect to the travel time between zone-pairs. In reality, however, the distribution of trip rates with respect to interzonal travel time is peaked at a positive value. For modeling purposes this nonmonotonic relationship must be estimated with a best fitting monotone form.

We specified and applied the following demand function. The function is a version of the gravity model (Wilson, 1970, Putnam, 1983), which explains changes in the way spatial activities interact,

$$q_{rs} = \frac{O_r \cdot D_s \cdot A_r \cdot B_s}{1 + \exp(\alpha + \beta \cdot u_{rs})} \quad (17)$$

where

- q_{rs} = trip rate between OD pair r - s ,
- u_{rs} = travel time between OD pair r - s ,
- O_r = trip production from origin zone r ,
- D_s = trip attraction to destination zone s ,
- A_r = origin zone r specific coefficient to be estimated,
- B_s = destination zone s specific coefficient to be estimated, and
- α, β = model parameters to be estimated.

The model coefficients, and parameters are estimated iteratively. The user equilibrium model estimates a matrix of zone-to-zone travel times [u_{rs}], based on a matrix [q_{rs}] describing OD requirements. An econometric model estimates the coefficients α, β from OD requirements and zone-to-zone travel times. Given travel times and estimated parameters, the gravity model is applied to estimate zone specific coefficients (A_r, B_s). Once initial estimates have been obtained for all unknowns, an updated set of OD requirements [q_{rs}] is estimated. These steps are repeated until the estimated values α, β stabilize. This procedure converges quickly.

A Medium-Scale Example: Applying the Variable Demand Model to the Memphis, Tennessee Network

Mississippi River detours. The representation of Memphis network was specified to introduce the option of a wide detour for traffic flowing into the region from West of Mississippi River. See Figure 2. If the Interstate Route 40 bridges across the Mississippi river are eliminated due to damage, travel demand detours and enters the region at several locations in the south of Memphis.

Three examples. A walk-through, Seismic Risk Analysis (SRA) of Memphis has been conducted by Werner, et al., (2000, 2003) for the Multidisciplinary Center for Earthquake Engineering Research (MCEER) and the Federal Highway Administration. The analysis applied the Frankel, et al., (1996) model for the Central United States to generate 2,321 earthquakes with moment magnitudes ranging from M 5.0 to M 8.0. From these, we selected three representative cases of network damage. These are cases CC40962, JJ9295, and JJ40962. For each case, network damage is assumed to be repaired incrementally; and in each of these three cases network configurations are reported for three points in time, seven days, 60 days, and 150 days following the earthquake.

These inputs were used to model network flows under assumptions of both fixed and variable travel demand. As shown in Table 1, the loss and restoration of network capacity has a substantial impact on system-wide travel times. In this exercise, the increases in travel time ranges from 39% to 145% depending on the realized level of damage and the amount of capacity that has been restored following the earthquake.

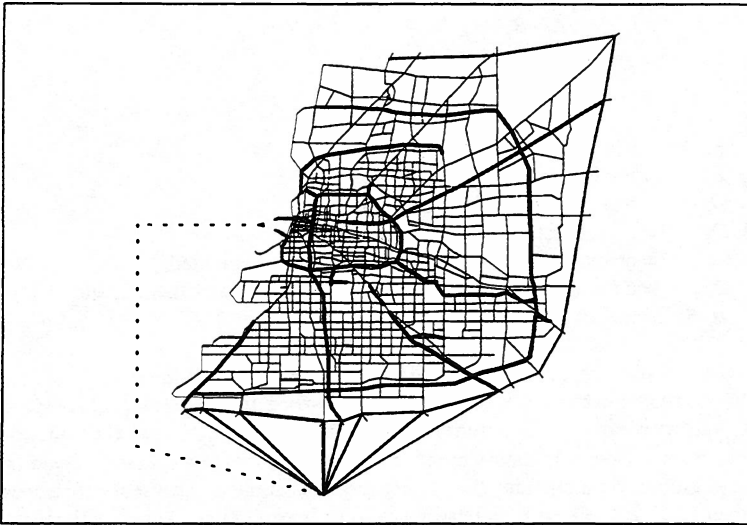


Figure 2. Memphis transportation network: Mississippi River detour scheme.

Table 1. Travel time estimates for the Memphis network: Fixed travel demand.

Senario	Total System Delay (PCU ^a • Hours)	Average Travel Time (minutes)	System Travel Demand (PCU)	
Baseline	15,134,460.43	18.748	4,978,307	
CC40962	7 days	42,414,441 (180.3%)	45.981 (+145.3%)	4,978,307
	60 days	29,862,091 (+97.3%)	32.210 (+71.8%)	4,978,307
	150 days	25,269,027 (+67.0%)	26.900 (+43.5%)	4,978,307
JJ9295	7 days	26,604,318 (+75.8%)	28.421 (+51.6%)	4,978,307
	60 days	25,553,932 (+68.8%)	27.150 (+44.8%)	4,978,307
	150 days	24,583,169 (+62.4%)	26.042 (+38.9%)	4,978,307
JJ40962	7 days	31,828,428 (+110.3%)	34.367 (+83.3%)	4,978,307
	60 days	28,200,775 (+86.3%)	30.305 (+61.6%)	4,978,307
	150 days	25,179,945 (+66.4%)	26.804 (+43.0%)	4,978,307

Note: a. Passenger Car Units. Freight and passenger flows are measured in terms of PCUs.

Table 2, in contrast, is based on an assumption of variable demand, and accounts for the way changes in network capacity induces changes in both travel time and trip rates. These effects both influence total system delay, but in opposing directions. Average travel times increase as network capacity is reduced. At the same time, the model predicts the travel demand (trip rates) will be reduced. These

net changes in system delay combine reductions in demand and increases in link travel times, and may be either positive or negative. In this exercise, total system delay is reduced, because travel is reduced.

Table 2. Travel time estimates for the Memphis network: Variable travel demand.

Scenario	System Cost (PCU ^a • Hours)	Average Travel time (minutes)	System Travel Demand (PCU)
Baseline	15,134,460	18.748	4,978,307
CC40962	7 days	13,146,626	21.555 (+15.0%)
	60 days	14,154,427	19.911 (+6.2%)
	150 days	14,814,855	19.146 (+2.1%)
JJ9092	7 days	14,559,738	19.499 (+4.0%)
	60 days	14,717,086	19.185 (+2.3%)
	150 days ^b	14,718,516	18.810 (+0.3%)
JJ40962	7 days	13,146,626	20.145 (+7.5%)
	60 days	14,154,427	19.767 (+5.4%)
	150 days	14,814,855	19.089 (+1.8%)
			4,302,625 (-13.6%)
			4,355,090 (-12.5%)
			4,774,186 (-4.1%)

Note: a. Passenger Car Units. Freight and passenger flows are measured in terms of PCUs.

b. Results required 100 iterations of the solution algorithm. All other examples required 25.

The variable demand model is intended to adjust OD requirements in response to changes in the level of service available on the network. These adjustments do not necessarily mean that the longest trips are eliminated first. The largest share of the trips eliminated by reductions in network level of service are trips ranging from 10 to 30 minutes in length. The relative trip rate reductions tend to be greatest for those zone pairs showing the greatest relative increase in travel times. Interzonal travel times range from 85% of baseline times to 200%. Increasing travel time by 20% results in travel demand that is about 80% of the baseline value.

Trips Forgone. Time has value. In most network flow models, cost and delay are treated as interchangeable quantities. This standard perspective is deficient in the context of this application. While an earthquake may well result in lower total system delay because few trips are taking place, the trips that have been forgone also have value, and their absence is not a cost accounted for by network delay costs.

Characterizing total transportation system cost in a variable demand context requires accounting for network congestion costs and the opportunity costs associated with trips forgone. Total system delay is the sum across all links of the total travel time accumulating on each of link. In addition to network delay, demand eliminated from the system due to increased congestion implies additional costs. This opportunity cost associated with these trips forgone is more difficult to calculate. The value of these trips is at least as great as the baseline delay associated with making them, otherwise these baseline trips would not occur. Consequently, the baseline

delays associated with trips forgone after the earthquake provide a useful lower bound on value of trips forgone.

The lower bounds on the total cost associated trips forgone are of the same order of magnitude as total congestion costs. When the opportunity cost of trips forgone is accounted for and combined with delay costs, the total daily transportation cost accumulating as a result of the earthquake ranges from \$311 million to \$358 million across the 4 cases. These costs are based on a unit value of time estimate that incorporates several value of assumptions for travelers and freight.

A Large-Scale Example: Applying the Variable Demand Model to the San Francisco Bay Area Network

The variable demand results associated with the Memphis application are encouraging, but Memphis is a relatively small city. Extending standard network equilibrium flow models to account variable demand in a much larger network presents a further challenge. The Metropolitan Transportation Commission (MTC) highway network model for the San Francisco Bay Area consists of 1,120 traffic analysis zones and 26,904 network links. The MTC also provides a 1998 matrix of OD requirements for this network model. The California Department of Transportation (Caltrans) District 4 provides a bridge inventory, which we combined with the MTC network as part of the Pacific Earthquake Engineering Research Center's (PEER) Highway Demonstration Project (Kiremidjian, et al. 2003).

The PEER Highway Demonstration Project examines four scenario events, including a moment magnitude $M 7.5$ earthquakes on the Hayward fault that is used here. The vulnerability of bridges to ground shaking, liquefaction and landslides is evaluated considering each hazard separately and in combination. Moderately and severely damaged bridges are assumed closed immediately after the earthquake. These results make it possible to calibrate and test the variable demand model on the Bay Area network, and to compare these results to network flow and delay estimates premised on fixed demand.

The cumulative link counts in Figure 3 summarize and compare the results for the fixed and variable demand models across all of the links in the network. Results providing volume/capacity (v/c) ratios above 1.0 indicate that network has been swamped by infeasibly large, unrealistic travel demands. In the fixed demand results, post-earthquake v/c ratios routinely exceed 1.0, in the most extreme examples by an order of magnitude. The network capacity losses associated with the earthquake are so extreme that any conventional network flow model is left with no substantive predictive power: The conditions being modeled are too extreme for the model to represent meaningfully. In contrast, the results for the variable demand formulation are much more realistic with respect to both traveler behavior and the technical performance of the network. Predicted volume/capacity ratios slightly exceed 1.0 in a few cases. Average ratios within link categories never do. As a result of the very substantial loss in network capacity associated with this event, congestion and total network delay costs increase in the variable demand case despite substantial reductions in travel demand. See Figure 4 for relative changes in trip origins across Bay Area traffic analysis zones.

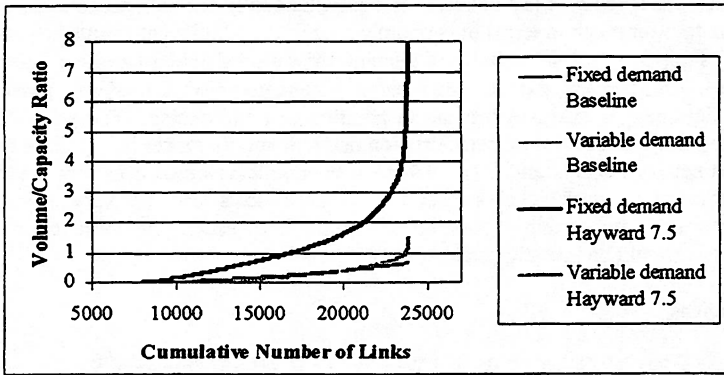


Figure 3: Comparison of v/c ratios predicted by the fixed demand and variable demand models for transportation flows associated with the Bay Area baseline and Hayward M 7.5 earthquake scenario.

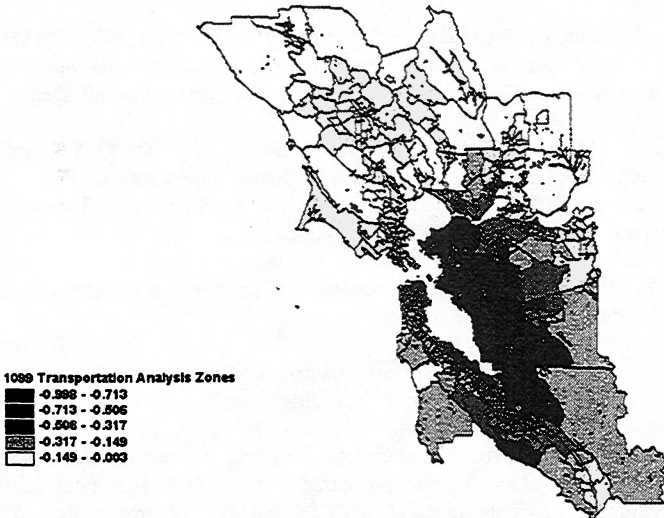


Figure 4: Relative differences across travel analysis zones in trip productions predicted by the variable demand model for the Hayward M 7.0 earthquake scenario.

Conclusions

Ideally, post-earthquake transportation flows would be model based on estimated damage to both the transportation network and the urban activity system (Cho, et al., 2000). This is computationally feasible, but challenging. The alternative

approach presented here is a convergent, computationally efficient approximation that appears to offer much in terms of economic meaning, and relevant results.

Transportation is an induced demand driven by demand for other goods and services. This research extends and integrates standard network modeling procedures to predict changes in travel demand in addition to route choice. The possibility of substantial, sudden losses of transportation network supply makes it important to look beyond network delays and account for the economic value of trips forgone due to decreases in network level of service. These costs can be very significant. Improvements are certainly possible with respect to estimating these opportunity costs, and should be investigated.

References

- Beckmann, M., McGuire, C. B., and Winsten, C. B. (1956) *Studies in the economics of transportation*, Yale University Press, New Haven.
- Cho, S.B., Gordon, P., Moore II, J.E., Richardson, H.W., Shinozuka, M., and Chang, S.E. (2001) "Integrating transportation network and regional economic models to estimate the costs of a large urban earthquake," *J. of Regional Sci.*, 41(1), 39-65.
- Frankel, A., Mueller, C., Barnhard, T., Perkins, D., Lyendecker, E.V., Dickman, N., Hanson, S., and Hopper, M. (1996) *National seismic hazard maps, documentation June 1996*, United States Geological Survey, Open-File Report 96-532, Denver.
- Kiremidjian, A., Moore II, J. E., Basoz, N., Williams, M., Fan, Y. Y., Yazlali, O., Roblee, C., and Wang, Z. (2003) "The PEER Highway Demonstration Project," paper presented at the ASCE Technical Council on Lifeline Earthquake Engineering's Sixth U.S. Conference, August 10-13, 2003, Long Beach.
- Putnam, S. H. (1983) *Integrated urban models: Policy analysis of transportation and land use*, Pion, Ltd., London.
- Sheffi, Y. (1985) *Urban transportation networks-equilibrium analysis with mathematical programming methods*, Prentice-Hall, Englewood Cliffs.
- Werner, S. D. (2003) "Current developments and future directions for seismic risk analysis of highway systems," paper presented at the ASCE Technical Council on Lifeline Earthquake Engineering's (TCLEE) Sixth U.S. Conference, August 10-13, 2003, Long Beach.
- Werner, S.D., Taylor, C.E., Moore, J.E. II, Walton J.S., and Cho, S.B. (2000) *A risk-based methodology for assessing the seismic performance of highway systems*, Report No. MCEER-00-0014, Multidisciplinary Center for Earthquake Engineering Research, Buffalo.
- Wilson, A.G. (1970) *Entropy in urban and regional modeling*, Pion Ltd., London.